FAST algorithm implementation

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1 FAST algorithm

1.1 Equivalent MIP formulation of FAST algorithm

FAST algorithm was first proposed in [1]. It finds an optimal solution to the unit commitment and economic dispatch (UCED) of a single-node energy system, where no transmission is considered. The problem to be solved is equivalent to the following MIP.

Table 1: Nomenclature of the original formulation

η_{th}	Thermal efficiency (%).
C_{avg}	Average cost (€/MWh).
C_{CO_2}	CO_2 costs (€/t_{CO_2}).
C_{FL}	Full-load cost (€/h).
C_{fuel}	Gross fuel costs (€/MWh_{fuel}).
C_{incr}	Incremental cost (€/MWh).
C_{NL}	No-load cost (€/h).
$C_{O\&M}$	O&M costs (€/MWh).
C_{start}	Cost of starting unit type (€/start).
f_{obj}	Annual costs (objective function).
G	Hourly costs.
I	Installed flexible generation (MW).
N	Number of inflexible generators (*integer*).
P_{dem}	System electrical demand (MW).
P_{max}	Maximum unit production (MW).
P_{min}	Minimum unit production (MW).
P_{res}	Minimum total reserve (MW).
P_{wind}	Available wind production (MW).
$R_{CO_2/MWh}$	Emissions Ratio (t_{CO_2}/MWh_{fuel}).
R_{NL}	No load ratio.
$t \in T$	Time-steps.
$u_{fl} \in U_{fl}$	Flexible generation types (e.g. single-cycle gas and distillate oil plant).
$u_{in} \in U_{in}$	Inflexible unit types (e.g. CCGT and steam-cycle Coal).
$u_{all} \in U_{all}$	All generation types.
V_{curt}	Decision variable: Wind curtailment (MW).
V_{gen}	Decision variable: Electrical output (MW).
V^b_{online}	Decision variable: Number online (*binary*).
V^b_{start}	Decision variable: Unit start (*binary*).

Objective function

Objective is to minimise total operational costs over the modelling period.

$$
\begin{aligned}
f_{obj} &= \sum_{t \in T} G(t) \\
G(t) &= \sum_{u_{in} \in U_{in}} \Big[C_{start}(u_{in}) \cdot V^b_{start}(u_{in}, t) + C_{NL}(u_{in}) \cdot V^b_{online}(u_{in}, t) + C_{incr}(u_{in}) \cdot V_{gen}(u_{in}, t) \Big] \\
&\quad + \sum_{u_{fl} \in U_{fl}} \Big[C_{avg}(u_{fl}) \cdot V_{gen}(u_{fl}, t) \Big], \; \forall t \in T
\end{aligned}
$$

Constraints

The constraints the operation (UCED) of the modelled system.

Total production of all units (less wind curtailment) must equal the electricity demand for each hour:

$$\sum_{u_{all} \in U_{all}} V_{gen}(u_{all}, t) + P_{wind}(t) - V_{curt}(t) = P_{dem}(t), \; \forall t \in T.$$

Wind curtailment must be less than or equal to wind production in each hour:

$$V_{curt}(t) \leq P_{wind}(t), \; \forall t \in T.$$

The sum of online inflexible capacity less the sum of inflexible production (i.e. the quantity of spinning reserve) must exceed a pre-defined target:

$$P_{res} \leq \sum_{u_{in} \in U_{in}} \Big[V^b_{online}(u_{in}, t) \cdot P_{max}(u_{in}) \Big] - \sum_{u_{in} \in U_{in}} \Big[V_{gen}(u_{in}, t) \Big], \; \forall t \in T.$$

For simplicity, here this target is set to the size of the largest installed unit, where at least one such unit exists.

Production from any type cannot exceed the quantity installed. Similarly, the number of online inflexible units cannot exceed the number installed:

$$
\begin{aligned}
\sum_{u_{all} \in U_{all}} V_{gen}(u_{all}, t) &\leq I(u_{all}), \; \forall t \in T \\
\sum_{u_{in} \in U_{in}} V^b_{online}(u_{in}, t) &\leq N(u_{in}), \; \forall t \in T.
\end{aligned}
$$

Production from any inflexible type is bounded by minimum and maximum output levels and the number of online units of that type:

$$
\begin{aligned}
V_{gen}(u_{in}, t) &\geq V^b_{online}(u_{in}, t) \cdot P_{min}(u_{in}), \; \forall t \in T, \; u_{in} \in U_{in} \\
V_{gen}(u_{in}, t) &\leq V^b_{online}(u_{in}, t) \cdot P_{max}(u_{in}), \; \forall t \in T, \; u_{in} \in U_{in}.
\end{aligned}
$$

Finally, the number of starts for an inflexible type is set to the increase in the number of online units for that type:

$$V^b_{start}(u_{in}, t) \geq V^b_{online}(u_{in}, t) - V^b_{online}(u_{in}, t-1), \, \forall t \in T, \, \forall u_{in} \in U_{in}.$$

In summary, the MIP is formulated as below:

$$\min_{\{V^b_{start}, V^b_{online}, V_{gen}\}} f_{obj} = \sum_{t \in T} G(t) \tag{1}$$

$$G(t) = \sum_{u_{in} \in U_{in}} \Big[C_{start}(u_{in}) \cdot V^b_{start}(u_{in}, t) + C_{NL}(u_{in}) \cdot V^b_{online}(u_{in}, t) + C_{incr}(u_{in}) \cdot V_{gen}(u_{in}, t) \Big]$$

$$+ \sum_{u_{fl} \in U_{fl}} \Big[C_{avg}(u_{fl}) \cdot V_{gen}(u_{fl}, t) \Big], \, \forall t \in T \tag{2}$$

$$\sum_{u_{all} \in U_{all}} V_{gen}(u_{all}, t) + P_{wind}(t) - V_{curt}(t) = P_{dem}(t), \, \forall t \in T \tag{3}$$

$$V_{curt}(t) \leq P_{wind}(t), \, \forall t \in T \tag{4}$$

$$P_{res} \leq \sum_{u_{in} \in U_{in}} \Big[V^b_{online}(u_{in}, t) \cdot P_{max}(u_{in}) \Big] - \sum_{u_{in} \in U_{in}} \Big[V_{gen}(u_{in}, t) \Big], \, \forall t \in T \tag{5}$$

$$\sum_{u_{all} \in U_{all}} V_{gen}(u_{all}, t) \leq I(u_{all}), \, \forall t \in T \tag{6}$$

$$\sum_{u_{in} \in U_{in}} V^b_{online}(u_{in}, t) \leq N(u_{in}), \, \forall t \in T \tag{7}$$

$$V_{gen}(u_{in}, t) \geq V^b_{online}(u_{in}, t) \cdot P_{min}(u_{in}), \, \forall t \in T, \, \forall u_{in} \in U_{in} \tag{8}$$

$$V_{gen}(u_{in}, t) \leq V^b_{online}(u_{in}, t) \cdot P_{max}(u_{in}), \, \forall t \in T, \, \forall u_{in} \in U_{in} \tag{9}$$

$$V^b_{start}(u_{in}, t) \geq V^b_{online}(u_{in}, t) - V^b_{online}(u_{in}, t-1), \, \forall t \in T, \, \forall u_{in} \in U_{in}. \tag{10}$$

1.2 Revision of the formulation

Sets and Indices

$u \in U^{fl}$	Set of flexible generation units (e.g. single-cycle gas turbine and distillate oil plant).
$u \in U^{in}$	Set of inflexible generation units (e.g. CCGT and steam-cycle coal generators).
$u \in U^{all}$	Set of all dispatchable generation units consisting of the flexible and the inflexible, i.e. $U^{all} = U^{fl} \cup U^{in}$.
$v \in V$	Set of variable renewable (VRE) generation units.
$t \in T$	Set of time-steps.

Parameters

η_u^{th}	Thermal efficiency (%) of dispatchable generation units at maximum production.
$C_{u:u\in U^{fl}}^{avg}$	Average cost (€/MWh) of flexible units.
C^{CO_2}	CO_2 costs (€/t_{CO_2}).
$C_{u:u\in U^{in}}^{FL}$	Full-load cost (€/h) of inflexible units.
$C_{u:u\in U^{in}}^{fuel}$	Gross fuel costs (€/MWh_{fuel}) of inflexible units.
$C_{u:u\in U^{in}}^{incr}$	Incremental cost (€/MWh) of inflexible units.
$C_{u:u\in U^{in}}^{NL}$	No-load cost (€/h) of inflexible units.
$C_u^{O\&M}$	O&M costs (€/MWh).
$C_{u:u\in U^{in}}^{start}$	Cost of starting unit type (€/start) of inflexible units.
C_t^{total}	Total system cost at individual time-step t.
$I_{U^{all}}$	Installed dispatchable generation (MW).
$N_{U^{in}}$	Number of installed inflexible generators (*integer*).
P_t^{dem}	System electrical demand (MW).
$P_{u:u\in U^{in}}^{max}$	Maximum unit production (MW) of inflexible units.
$P_{u:u\in U^{in}}^{min}$	Minimum unit production (MW) of inflexible units.
P_t^{res}	Minimum total reserve (MW).
$P_{v,t}^{avail}$	Available production (MW) of the variable renewables.
$R_u^{CO_2/MWh}$	Emissions Ratio (t_{CO_2}/MWh_{fuel}).
$R_{u:u\in U^{in}}^{NL}$	No load ratio of inflexible units.

$x^{curt}_{v,t}$	Curtailment (MW) of VRE units.
$x^{gen}_{u,t}$	Electrical output (MW) of dispatchable units.
$z^{online}_{u,t:u\in U^{in}} \in \{0,1\}$	Online decision (*binary*) of inflexible units.
$z^{start}_{u,t:u\in U^{in}} \in \{0,1\}$	Start up decision (*binary*) of inflexible units.

Variables

Formulation

Objective function:

$$\min_{\{z^{start}_{u,t:u\in U^{in}}, z^{online}_{u,t:u\in U^{in}}, x^{gen}_{u,t}\}} \sum_{t\in T} C^{total}_t, \tag{11}$$

where,

$$C^{total}_t = \sum_{u\in U^{in}} \left(C^{start}_u \cdot z^{start}_{u,t} + C^{NL}_u \cdot z^{online}_{u,t} + C^{incr}_u \cdot x^{gen}_{u,t}\right) + \sum_{u\in U^{fl}} C^{avg}_u \cdot x^{gen}_{u,t}, \forall t \in T.$$

Constraints:

$$\sum_{u\in U^{all}} x^{gen}_{u,t} + \sum_{v\in V} \left(P^{avail}_{v,t} - x^{curt}_{v,t}\right) = P^{dem}_t, \forall t \in T \tag{12}$$

$$x^{curt}_{v,t} \leq P^{avail}_{v,t}, \forall v \in V, \forall t \in T \tag{13}$$

$$\sum_{u\in U^{in}} \left(z^{online}_{u,t} \cdot P^{max}_u\right) - \sum_{u\in U^{in}} x^{gen}_{u,t} \geq P^{res}_t, \forall t \in T \tag{14}$$

$$\sum_{u\in U^{all}} x^{gen}_{u,t} \leq I_{U^{all}}, \forall t \in T \tag{15}$$

$$\sum_{u\in U^{in}} z^{online}_{u,t} \leq N_{U^{in}}, \forall t \in T \tag{16}$$

$$x^{gen}_{u,t} \geq z^{online}_{u,t} \cdot P^{min}_u, \forall u \in U^{in}, \forall t \in T \tag{17}$$

$$x^{gen}_{u,t} \leq z^{online}_{u,t} \cdot P^{max}_u, \forall u \in U^{in}, \forall t \in T \tag{18}$$

$$z^{start}_{u,t} \geq z^{online}_{u,t} - z^{online}_{u,t-1}, \forall u \in U^{in}, \forall t \in T. \tag{19}$$

And the incremental cost of inflexible generation units is estimated by:

$$C^{incr}_u = \frac{C^{FL}_u \cdot (1 - R^{NL}_u)}{P^{max}_u}, \tag{20}$$

where,

$$C^{FL}_u = \frac{P^{max}_u}{\eta^{th}_u} \cdot (C^{fuel}_u + C^{O\&M}_u + C^{CO_2} \cdot R^{CO_2/MWh}_u), \forall u \in U^{in}. \tag{21}$$

References

[1] Aonghus Shortt and Mark O'Malley. Quantifying the long-term impact of electric vehicles on the generation portfolio. *IEEE Transactions on Smart Grid*, 5(1):71–83, 2014.